One-Time Pad or Vernam Cipher

• The one-time pad, which is a provably secure cryptosystem, was developed by Gilbert Vernam in 1918.
• The message is represented as a binary string (a sequence of 0's and 1's using a coding mechanism such as ASCII coding.
• The key is a truly random sequence of 0's and 1's of the same length as the message.
• The encryption is done by adding the key to the message modulo 2, bit by bit. This process is often called *exclusive or*, and is denoted by *XOR*. The symbol $\oplus$ is used.

a	b	$c = a \oplus b$
0	0	0
0	1	1
1	0	1
1	1	0

One-Time Pad or Vernam Cipher

Example: Let the message be IF then its ASCII code be (1001001 1000110) and the key be (1010110 0110001).
The ciphertext can be found exoring message and key bits

Encryption:

1001001 1000110	plaintext
1010110 0110001	key
0011111 1110110	ciphertext = (v)

Decryption:

0011111 1110110	ciphertext
1010110 0110001	key
1001001 1000110	plaintext

Why One-Time Pad is provably secure?

Or how can we prove it is unbreakable?

- The security depends on the randomness of the key.
- It is hard to define randomness.
- In cryptographic context, we seek two fundamental properties in a binary random key sequence:
  1. *Unpredictability:* Independent of the number of the bits of a sequence observed, the probability of guessing the next bit is not better than ½. Therefore, the probability of a certain bit being 1 or 0 is exactly equal to ½.
  2. *Balanced (Equal Distribution):* The number of 1's and 0's should be equal.

Mathematical Proof

- the probability of a key bit being 1 or 0 is exactly equal to ½.
- The plaintext bits are not balanced. Let the probability of 0 be x and then the probability of 1 turns out to be $1-x$.
- Let us calculate the probability of ciphertext bits.

m_i	prob.	k_i	prob.	c_i	prob.
0	x	0	½	0	½ x
0	x	1	½	1	½ x
1	$1-x$	0	½	1	½ $(1-x)$
1	$1-x$	1	½	0	½ $(1-x)$

- We find out the probability of a ciphertext bit being 1 or 0 is equal to $(½)x + (½)(1-x) = ½$. Ciphertext looks like a random sequence.

A Practical One-Time Pad

- A satellite produces and broadcasts several random sequences of bit at a rate fast enough such that no computer can store more than a very small fraction of them.
- Alice & Bob use a PKC to agree on a method of sampling bits from these random sequences.
- They use these bits to form a key for one-time pad.
- Eve, in theory, can break the PKC they used even though doing so is difficult.
- But by the time she breaks it, random bits Alice & Bob collected disappeared and Eve can not decrypt the message since she hasn't got the resources to store all the random bits that have been broadcast.

- Symmetric-key ciphers
- Encrypt individual characters at a time,
- Faster and less complex in hardware,
- They are desirable in some applications in which
    - buffering is limited
    - bits must be individually processed as they are received.
    - Transmission errors are highly probable.
- Vast amount of theoretical knowledge.
- Various design principles.
- Widely being used at present, will probably be used in the future.

Basic Idea comes from **One-Time-Pad** cipher,

Encryption : $c_i = m_i \oplus k_i \qquad i = 1,2,3,\ldots$

m_i : plain-text bits.

k_i : key (key-stream) bits

c_i : cipher-text bits.

Decryption : $m_i = c_i \oplus k_i \qquad i = 1,2,3,\ldots$

- Provably Secure.

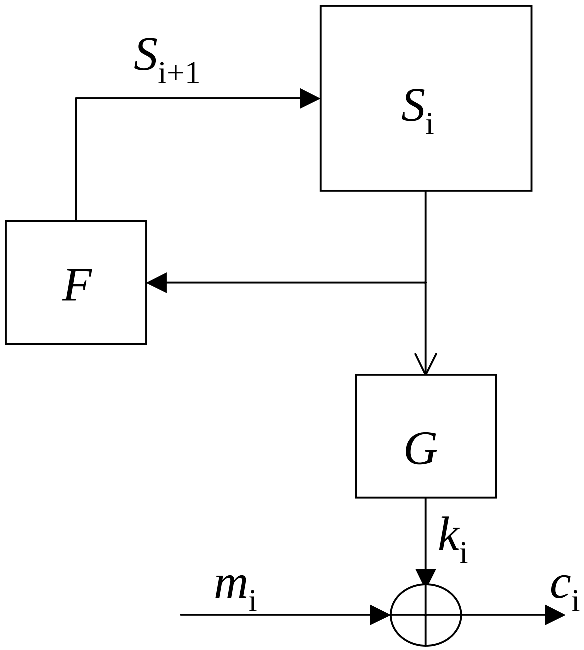
Drawback : Key-stream should be as long as plain-text.
Key distribution & Management difficult.

Solution : Stream Ciphers (in which key-stream is generated in pseudo-random fashion from relatively short *secret key*.

Randomness : Closely related to *unpredictability*.

Pseudo-randomness :PR sequences appears random to a computationally bounded adversary.

• Stream Ciphers can be modeled as Finite-state machine.



S_i : state of the cipher at time $t = i$.

F : state function.

G : output function.

Initial state, output and state functions are controlled by the secret key.

1.**Synchronous Stream Ciphers**

- Key-stream is independent of plain and cipher-text.
- Both sender &receiver must be synchronized.
- Resynchronization can be needed.
- No error propagation.
- Active attacks can easily be detected.

2. Self-Synchronizing Stream Ciphers

- Key-stream is a function of fixed number *t* of cipher-text bits.
- Limited error propagation (up to *t* bits).
- Active attacks cannot be detected.
- At most *t* bits later, it resynchronizes itself when synchronization is lost.
- It helps to diffuse plain-text statistics.

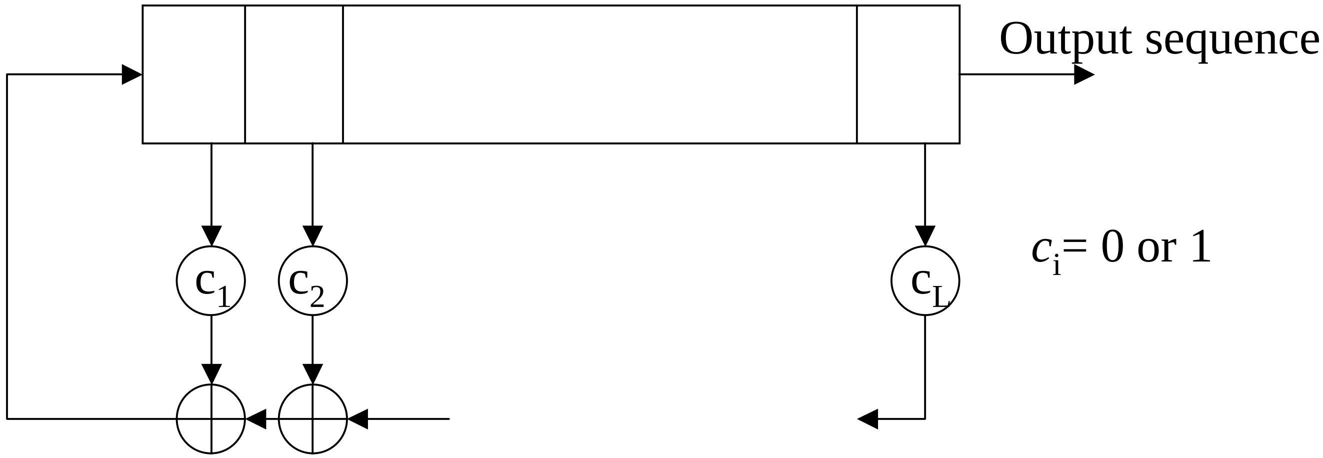
Analysis

• Efforts to evaluate the security of stream ciphers.

1. Mathematical Analysis
    - Period and Linear Complexity,
    - Security against Correlation Attacks.
2. Pseudo-randomness Testing
    - Statistical Tests,
    - Linear Complexity,
    - Ziv-Lempel Complexity
    - Maximum Order Complexity,
    - Maurer's Universal Test.

• In testing, all the tests are applied to as many key-streams of different lengths as possible.

Linear Feedback Shift Register - LFSR



$$C(x) = 1 + c_1 x + c_2 x^2 + \Lambda + c_L x^L$$: Connection Polynomial

- If $C(x)$ is primitive, LFSR is called *maximum-length*, and the output sequence is called *m-sequence* and its period is $T = 2^L$-1.
- *m-sequences* have good statistical properties.
- However, they are predictable.

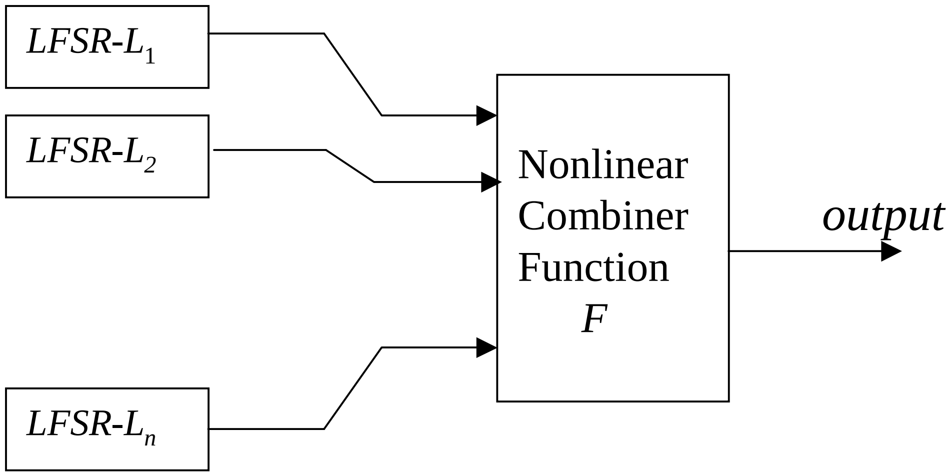
• If 2L successive bits of an m-sequence are known, the shortest LFSR which produces the rest of the sequence can be found using Berlekamp-Massey (BM) algorithm.
• Generally, the length of the shortest LFSR which generates a sequence is called linear complexity.

Stream Cipher Designs Based on LFSRs

• LFSRs generate m-sequence.
• However, " Linearity is the curse of cryptographer."
• The methods of utilizing LFSRs as *building blocks* in the stream cipher design.
• The design principle :

Use other blocks which introduce non-linearity while preserving the statistical properties of *m*-sequences.

Nonlinear combination Generators



The Combiner Function should be,

1. Balanced,
2. Highly nonlinear,
3. Correlation Immune.

• Utilizing the *algebraic normal form* of the combiner function we can compute the linear complexity of the output sequence.

Example (Geffe Generator) :

$$F(x_1, x_2, x_3) = x_1 x_2 \oplus x_2 x_3 \oplus x_3$$

If the lengths of the LFSRs are relatively prime and all connection polynomials are primitive, then

$$L = L_1 L_2 + L_2 L_3 + L_3$$

$$T = (2^{L_1} - 1) \cdot (2^{L_2} - 1) \cdot (2^{L_3} - 1)$$

When we inspect the truth table of the combiner function we gain more insight about the security of Geffe generator.

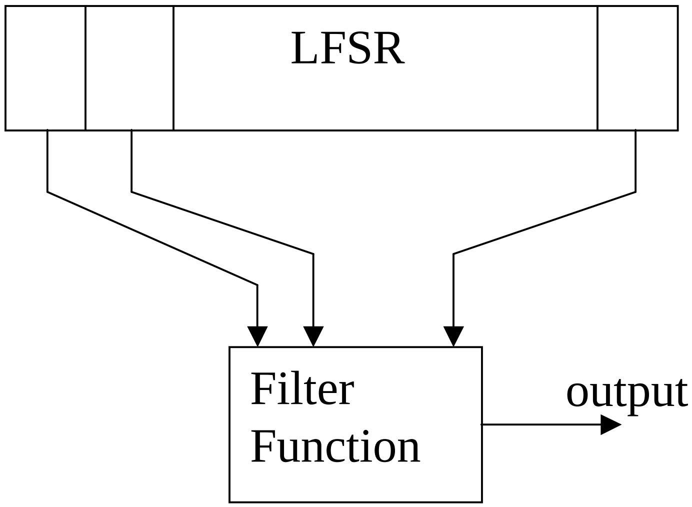
x_1	x_2	x_3	$z = F(x_1,x_2,x_3)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

- The combiner function is balanced.
- However, the correlation probability,

$$P(z = x_1) = 3/4.$$

- Geffe generator is not secure.

Nonlinear Filter Generator



- Upper bound for linear complexity,

$$L_m = \sum_{i=1}^{m} \binom{L}{i}$$

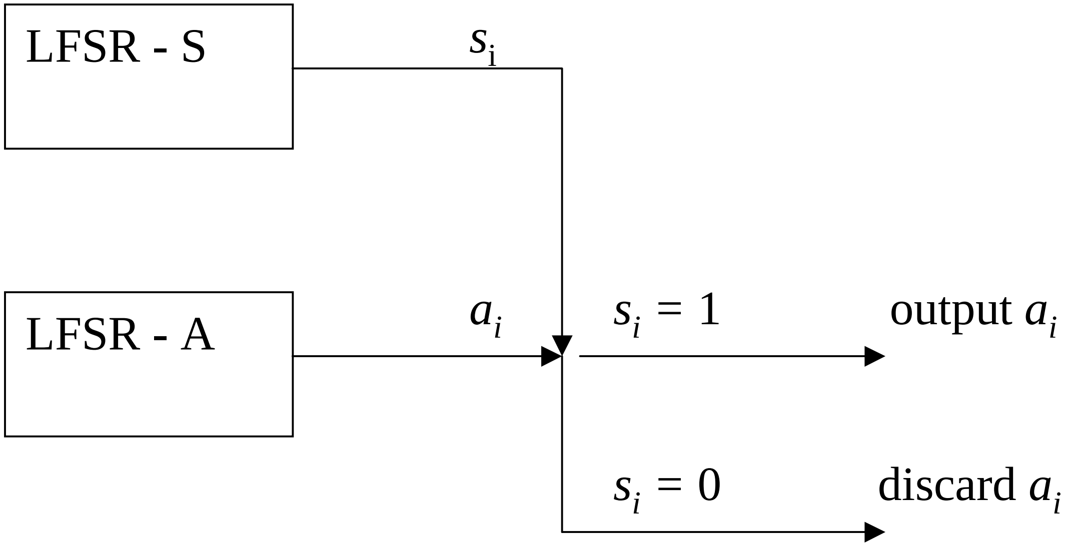
m : nonlinear order of the filter function.

- When L and m are big enough, the linear complexity will become large.

Clock-controlled Generators

• An LFSR can be clocked by the output of another LFSR.
• This introduces an irregularity in clocking of the first LFSR, hence increase the linear complexity of its output.

Example : Shrinking Generator



- Relatively new design.
- However, it is analyzed and it seems secure under certain circumstances.

$$if\ \gcd(L_s, L_A) = 1 \Rightarrow$$

$$T = (2^{L_A} - 1) \cdot 2^{L_S - 1}$$

$$L_A \cdot 2^{L_S - 2} < L < L_A \cdot 2^{L_S - 1}$$

Different Designs

- SEAL, RC4.
    - They use expanded key tables,
    - Fast in software,
    - Look secure,
    - They have not been fully analyzed yet,
    - Efficient analysis tools are not developed.